BITS Pilani
K K Birla Goa Campus

MECHANICS OF SOLIDS (ME F211)

VIKAS CHAUDHARI

Chapter-6

Torsion

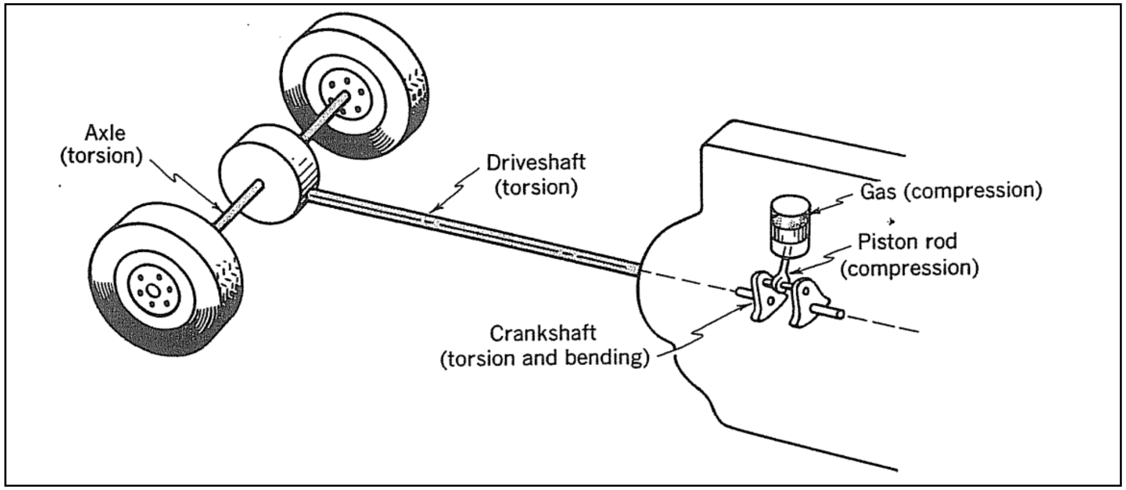
Torsion

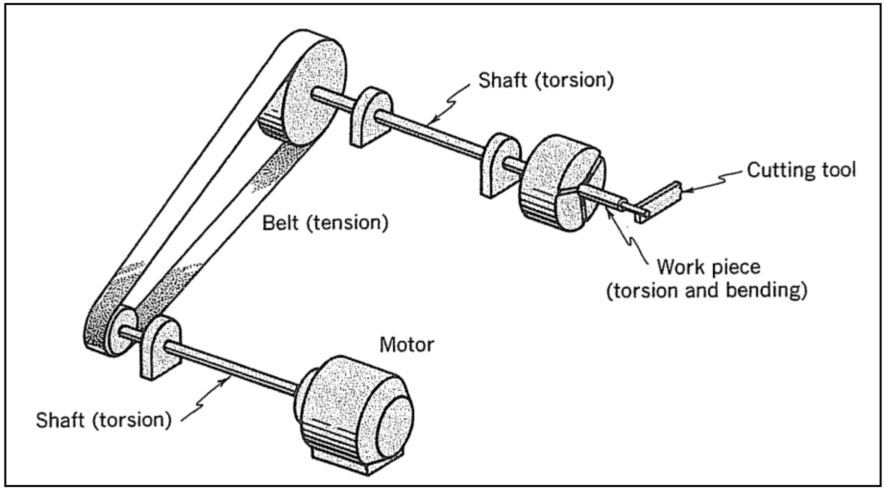
Introduction

- In the transmission of power by a shaft in torsion we are interested in twisting moment that can be transmitted by the shaft without damage to the material and hence we wish to know what the stresses in the shaft.
- Shaft is a Torsion spring
- We are interested in the relationship between applied twisting moment and resulting angular twist in the shaft.
- In order to obtain this overall force deformation relation, we shall have to consider the distribution of stress and strain throughout the entire member.

Torsion

Examples

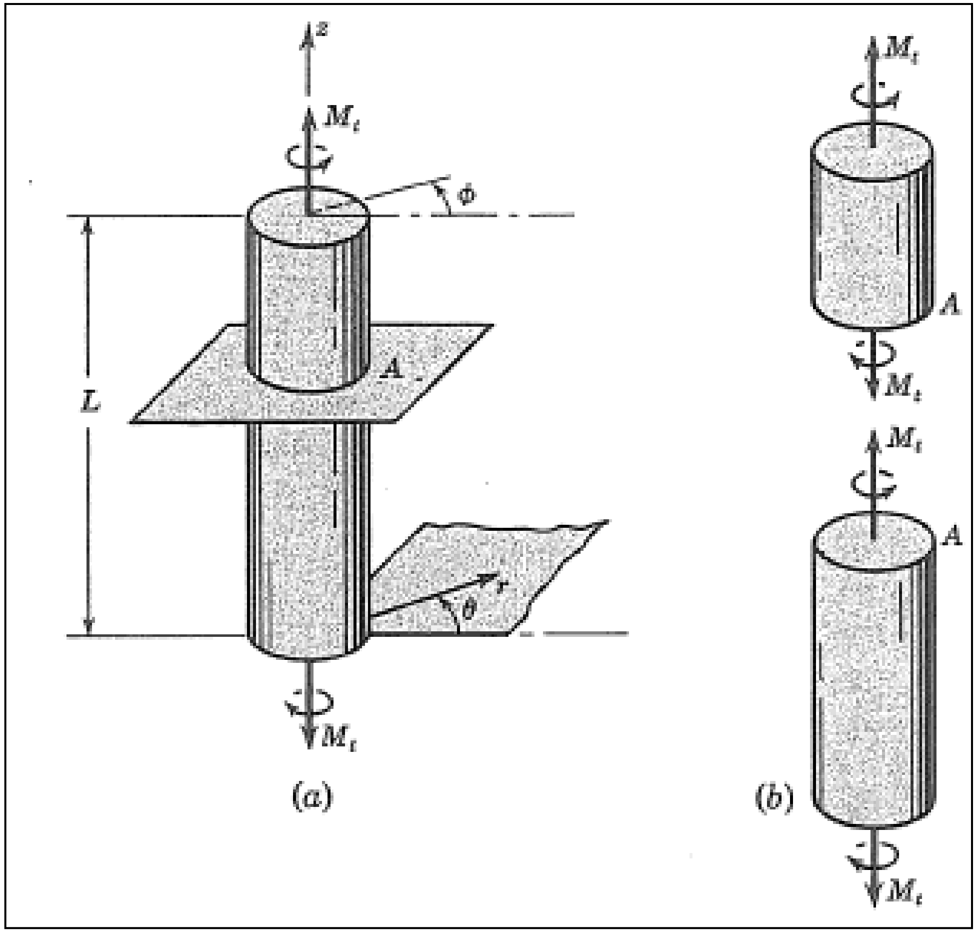




Steps for Solving Torsion Problem

- Examine the geometric behavior of twisted shaft
- Construct the plausible model for the deformation
- Stress strain relations are incorporated
- The conditions of equilibrium are applied

Geometry of Deformation of a Twisted Circular Shaft



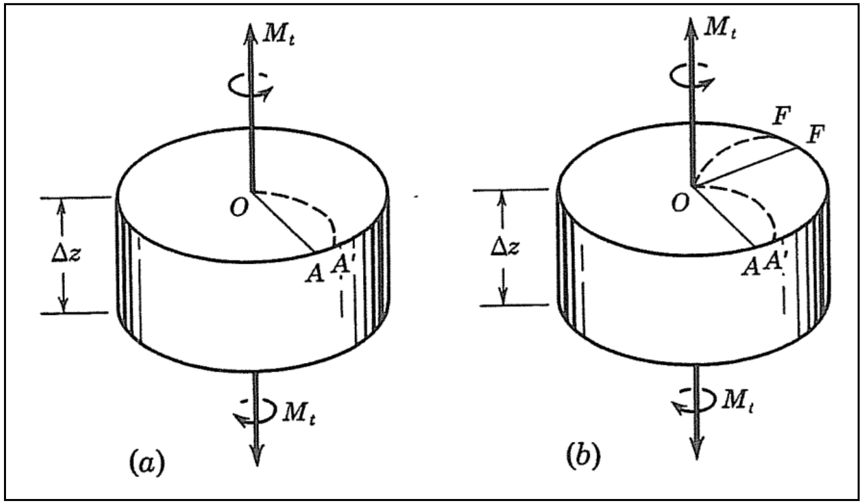
Geometry of Deformation of a Twisted Circular Shaft

Assumptions considered in the analysis of torsion problem

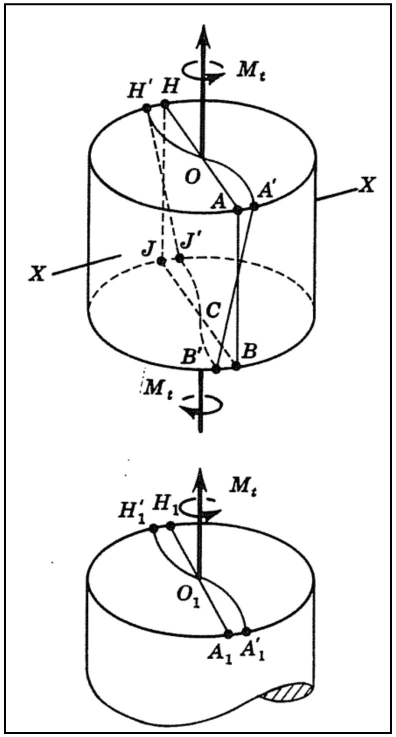
- The material used for the circular shaft is uniform homogenous and isotropic
- The C/S of circular shaft which were plane before twisting remains plane after twisting.
- The circular shaft must deform such that each plane C/S originally normal to the axis remains plane and normal
- The plane C/S does not distort within its own plane
- The amount of twist is assumed small and Extensional strains are zero i.e. $\varepsilon_z = \varepsilon_r = \varepsilon_\theta = 0$

Geometry of Deformation of a Twisted Circular Shaft

Assumptions considered in the analysis of torsion problem



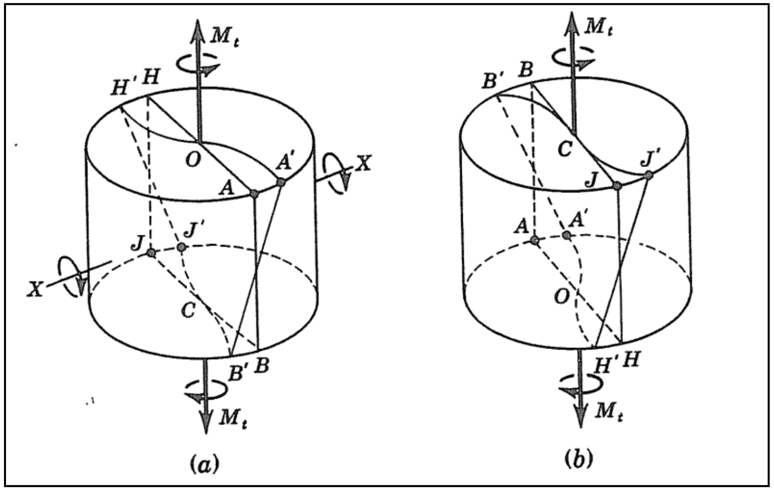
Thin slice showing hypothetical deformation

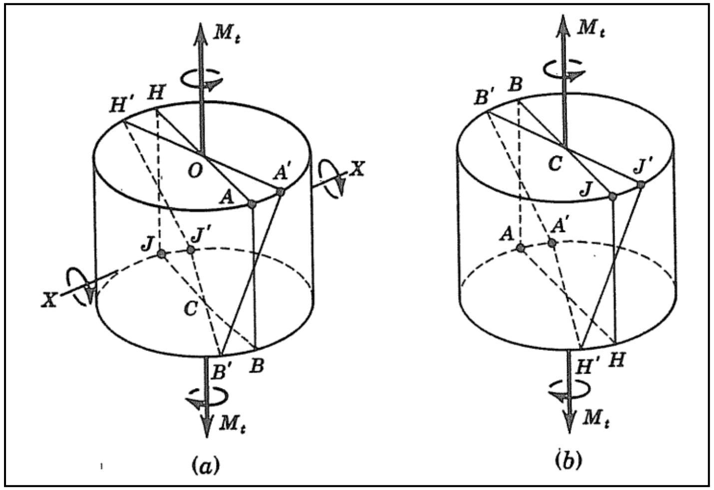


The assumed shape of *B'C'J'* does not match $A_1'O_1H_1'$

Geometry of Deformation of a Twisted Circular Shaft

Assumptions considered in the analysis of torsion problem





Rotating (*a*) about *X-X* through 180° yields (*b*), which has undergone different deformation even though the twisting moment and geometry are the same.

If diameter *HA* remains straight during deformation, then rotation of (*a*) about *X-X* produces (*b*) which is identical in terms of deformation.

Torsion

Geometric Compatibility

$$\gamma_{\theta z} = \lim_{\Delta z \to 0} \frac{E_0 E_1}{H_1 E_0} = \lim_{\Delta z \to 0} \frac{r\Delta\phi}{\Delta z} = r\frac{d\phi}{dz}$$

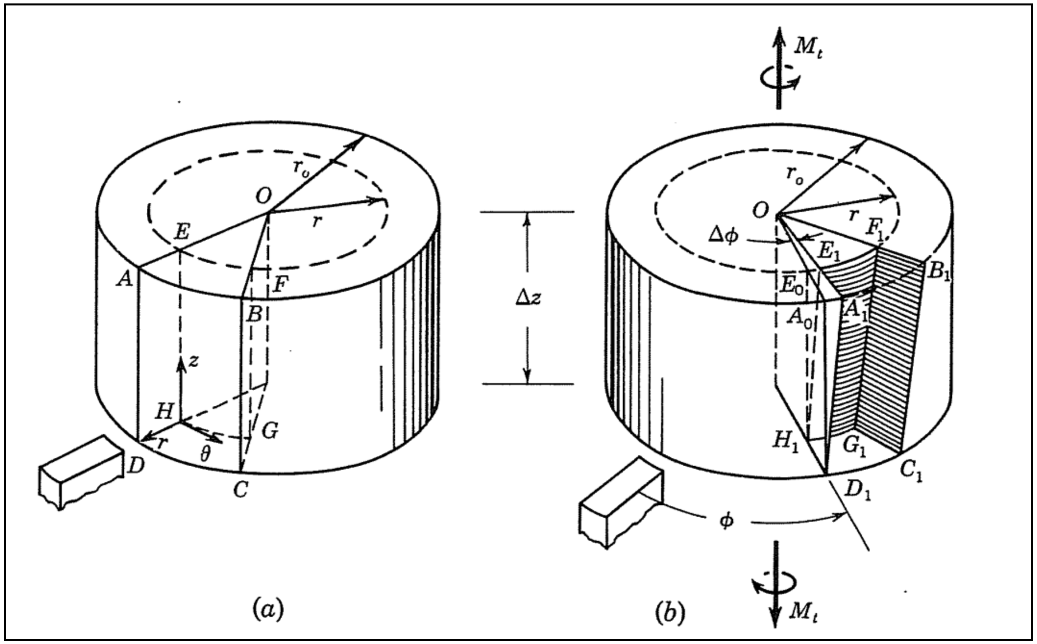
Stress-Strain Relation

$$\varepsilon_z = \varepsilon_r = \varepsilon_\theta = \gamma_{r\theta} = \gamma_{rz} = 0$$

$$\sigma_z = \sigma_r = \sigma_\theta = \tau_{r\theta} = \tau_{rz} = 0$$

$$\gamma_{\theta z} = r\frac{d\phi}{dz}$$

$$\tau_{\theta z} = G\gamma_{\theta z} = Gr\frac{d\phi}{dz}$$



Analysis of deformation of a slice of circular shaft subjected to torsion.

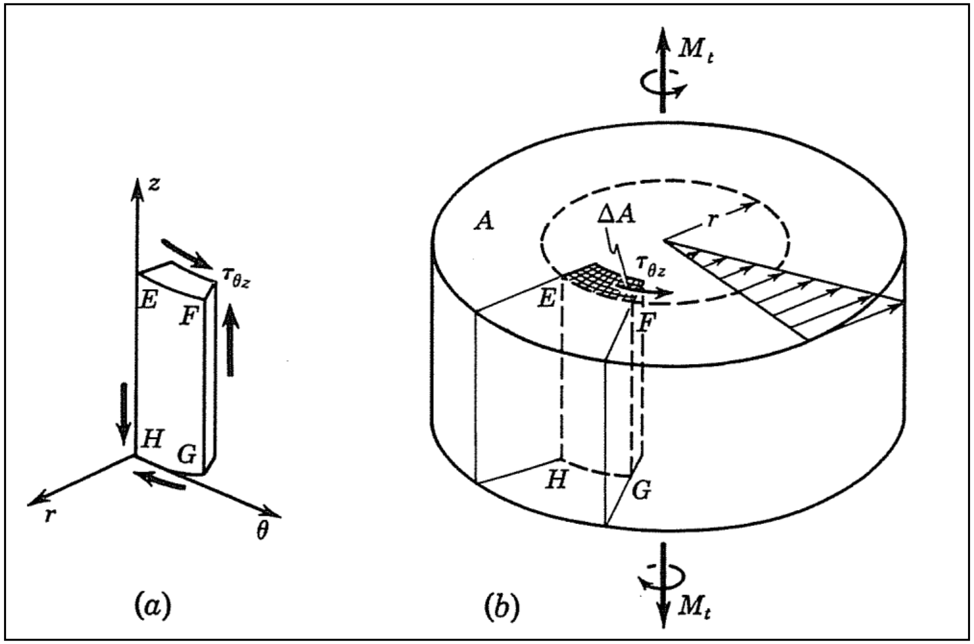
Equilibrium Requirements

$$\int_A r\left(\tau_{\theta z} dA\right) = M_t$$

$$M_t = G\frac{d\phi}{dz}\int_A r^2 dA = G\frac{d\phi}{dz} I_z$$

I_z is called polar moment of inertia

$$I_z = \frac{\pi r_0^4}{2} = \frac{\pi d^4}{32}$$



(*a*) Stress components acting on a small element; (*b*) distribution of shearing stress on cross section.

Equilibrium Requirements

Therefore the rate of twist in terms of applied twisting moment is

$$G\frac{d\phi}{dz} = \frac{M_t}{I_z}$$

By integrating above equation over the length of the shaft

$$\phi = \int_0^L \frac{M_t}{GI_z} dz = \frac{M_t L}{GI_z}$$

and

$$\frac{M_t}{\phi} = \frac{GI_z}{L}$$

is torsional stiffness

Torsion formula

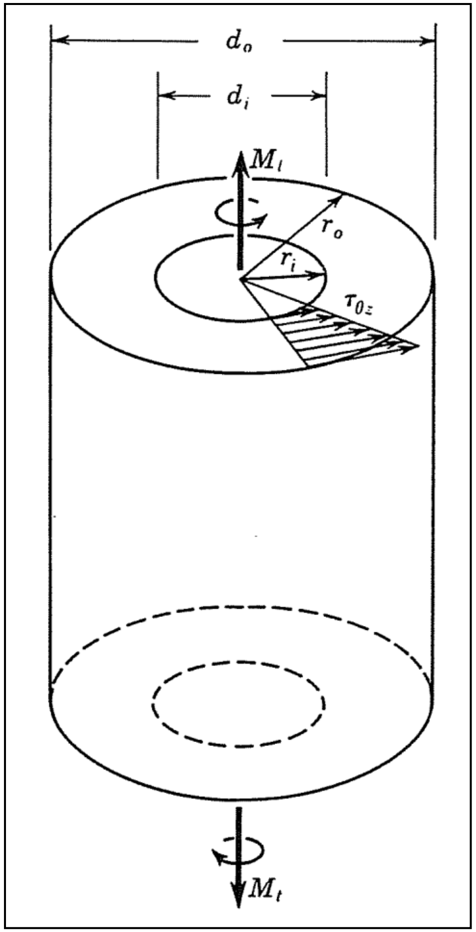
$$\frac{\tau_{\theta z}}{r} = \frac{G\phi}{L} = \frac{M_t}{I_z}$$

innovate achieve lead

Torsion of Elastic Hollow Circular Shafts

- The assumptions made for elastic solid circular shaft are valid for elastic hollow circular shaft.
- The only difference is that the integral in equilibrium equation now extends over the annulus instead of a complete circle
- Thus for hollow circular shafts, the polar moment of inertia is taken as

$$I_z = \frac{\pi}{2}\left(r_0^{\,4} - ri^4\right) = \frac{\pi}{32}\left(d_0^{\,4} - d_i^{\,4}\right)$$



Stress distribution in elastic hollow circular shaft

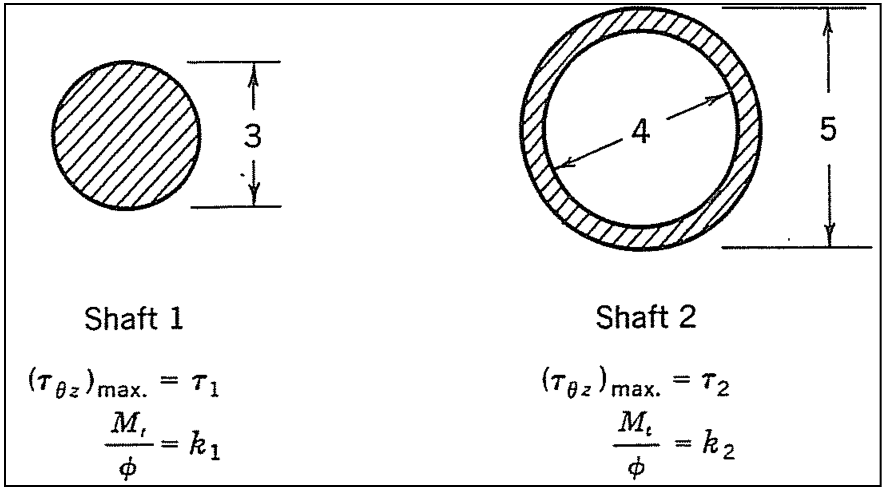
Torsion of Elastic Hollow Circular Shafts

Solid Shaft *Versus* Hollow Circular Shaft

When both shafts are twisted by same twisting moment,

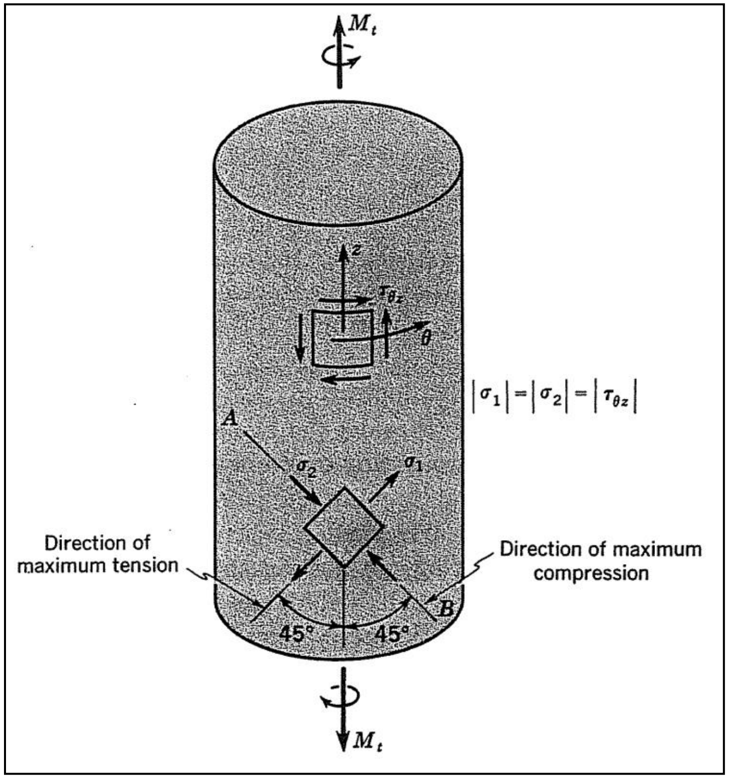
$$\text{Stress ratio} = \frac{\tau_2}{\tau_1} = \frac{15}{41} = 0.37$$

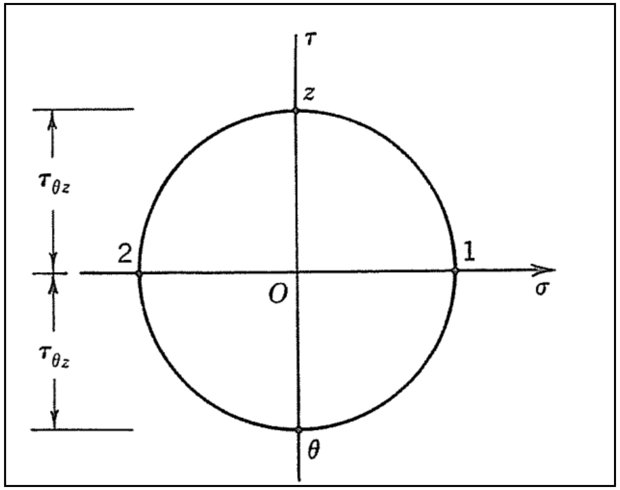
$$\text{Stiffness ratio} = \frac{k_2}{k_1} = \frac{41}{9} = 4.56$$



Torsion

Stress analysis in torsion; combined stresses



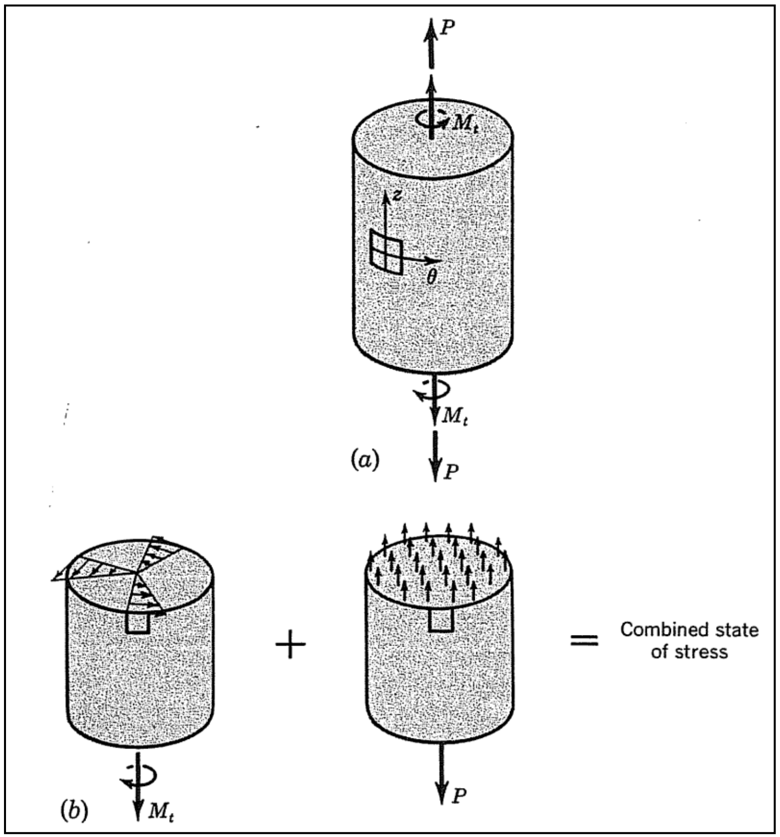


Mohr's circle for stress for element of shaft in torsion

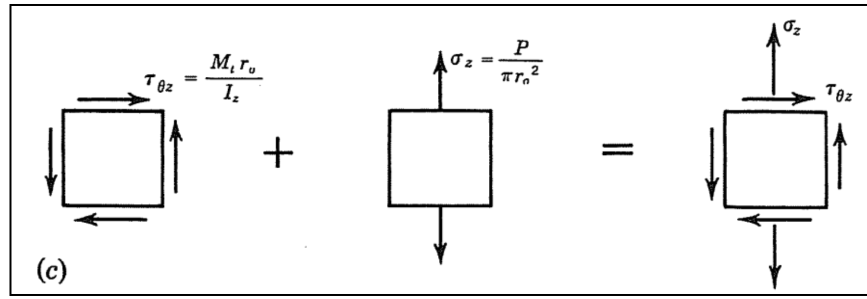
The principal stresses in torsion are equal tension and compression acting on faces inclined at 45° to the axis of the shaft

Torsion

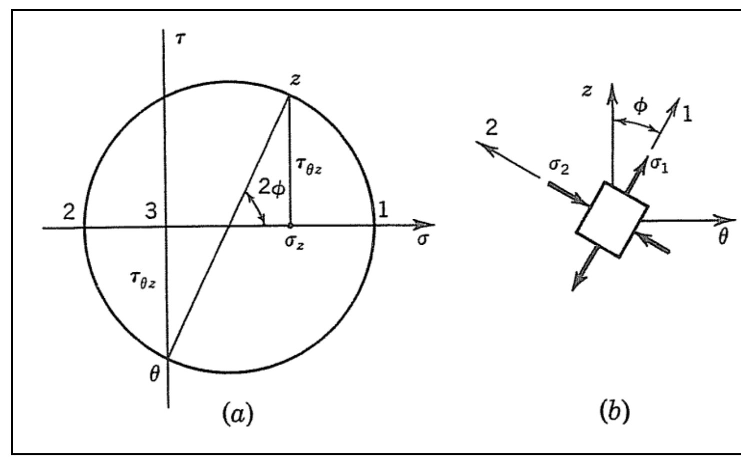
Stress analysis in torsion; combined stresses



Combined stresses due to torsion and tension



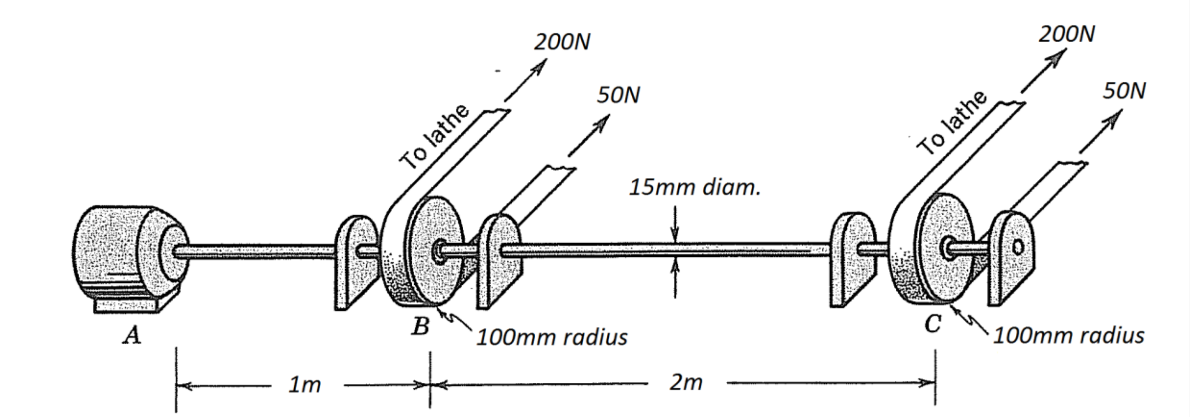
Combined stresses due to torsion and tension



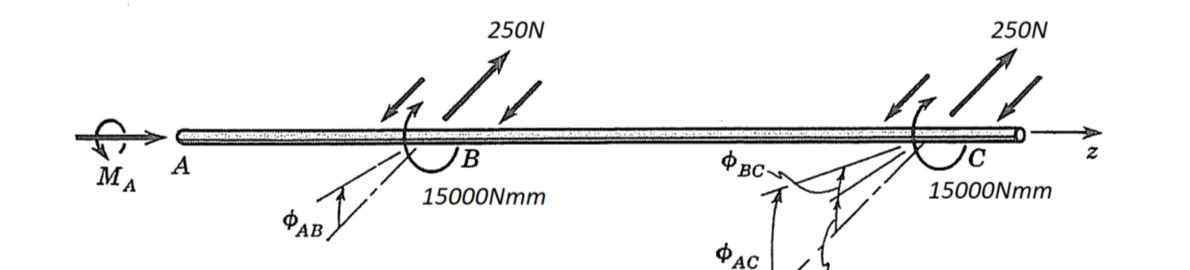
Principal directions and principal stresses

Example

Two small lathes are driven by the same motor through a 15 mm diameter steel shaft, as shown in Fig. below. We wish to know the maximum shear stress in the shaft due to twisting and the angle of twist between the two ends of the shaft.



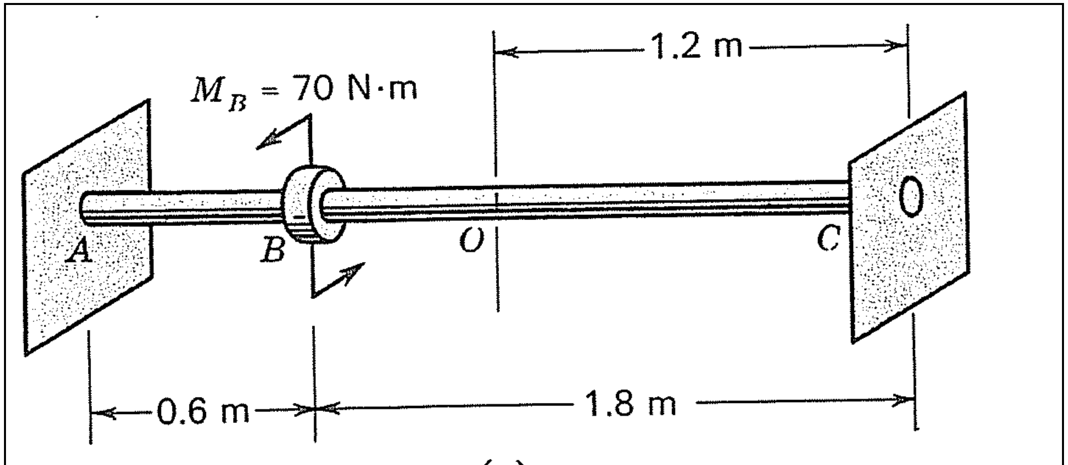
(a)



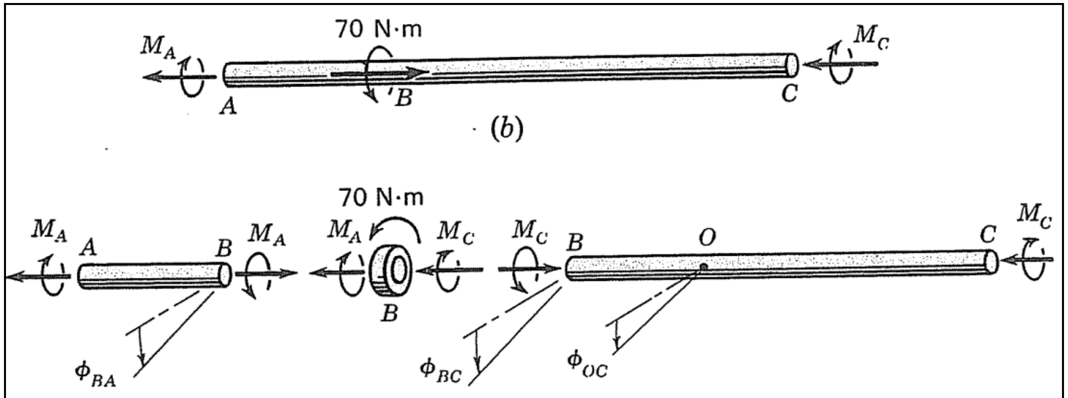
(b)

Example

A couple of 70Nm is applied to a 25mm diameter 2024-0 aluminum-alloy shaft, as shown in figure. The ends *A* and *C* of the shaft are built-in and prevented from rotating, and we wish to know the angle through which the center cross section *O* of the shaft rotates.



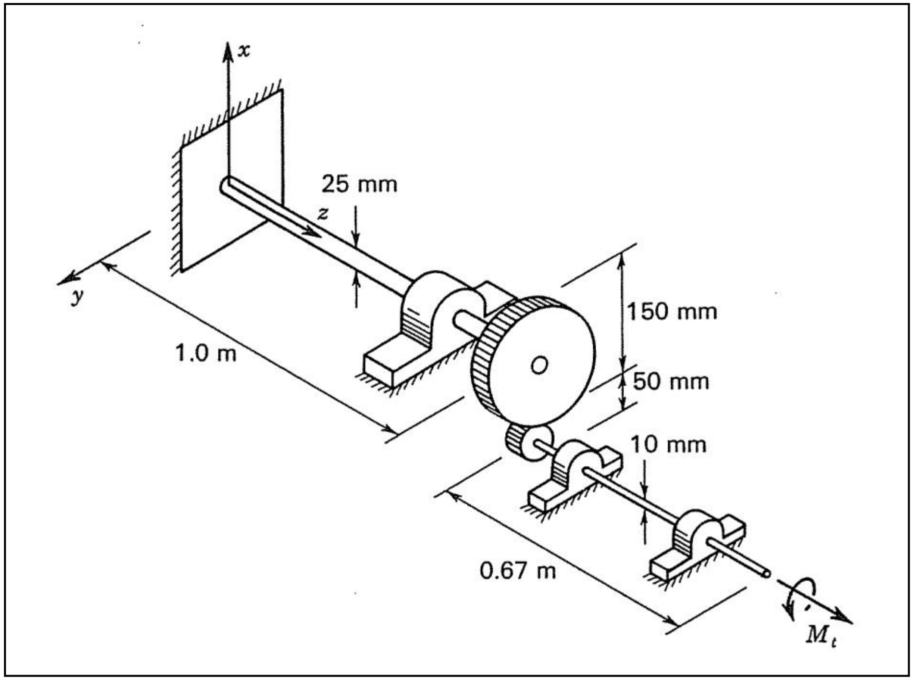
(a)



(c)

Problem

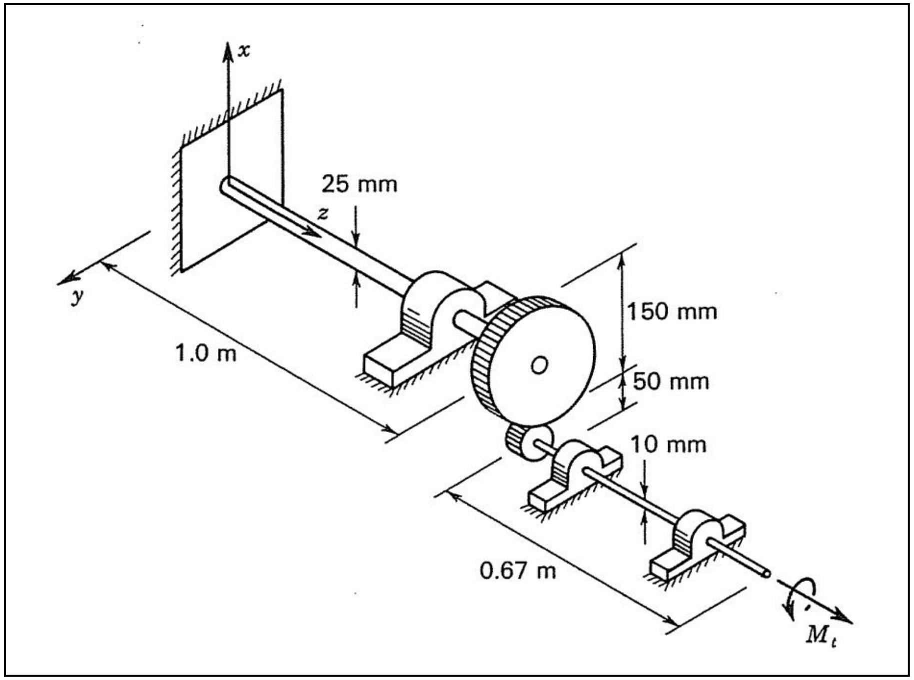
A torque M_t of 10Nm is applied as shown to the steel shafts geared together. Calculate the angle of twist at the point where the torque is applied.



Torsion

Problem

For the gear system shown below, what maximum torque M_t may be applied before a shear stress of 275MPa is reached in either shaft?

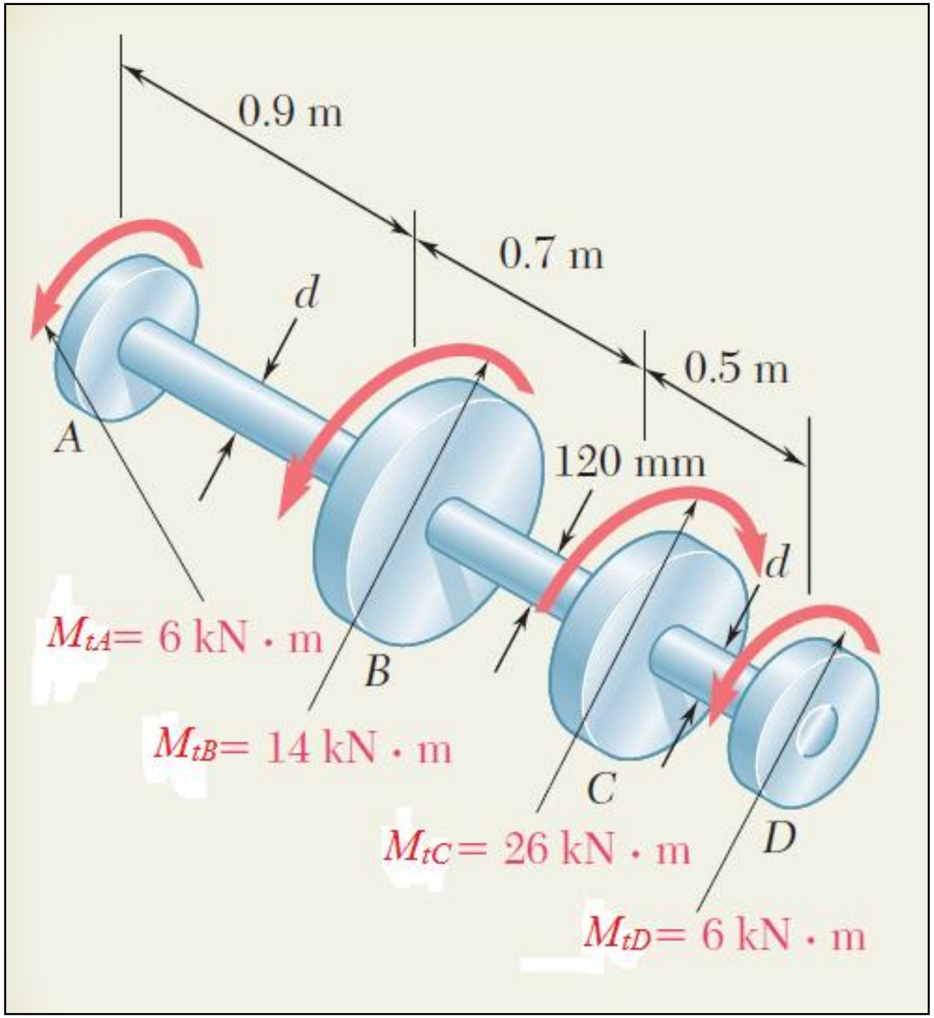


Problem

Shaft *BC* is hollow with inner and outer diameters of 90mm and 120mm, respectively. Shafts *AB* and *CD* are solid and of diameter *d*. For the loading shown, determine

(a) the maximum shearing stress in shaft *BC*.

(b) the required diameter *d* of the shafts *AB* and *CD* if the allowable shearing stress in these shafts is 65MPa.



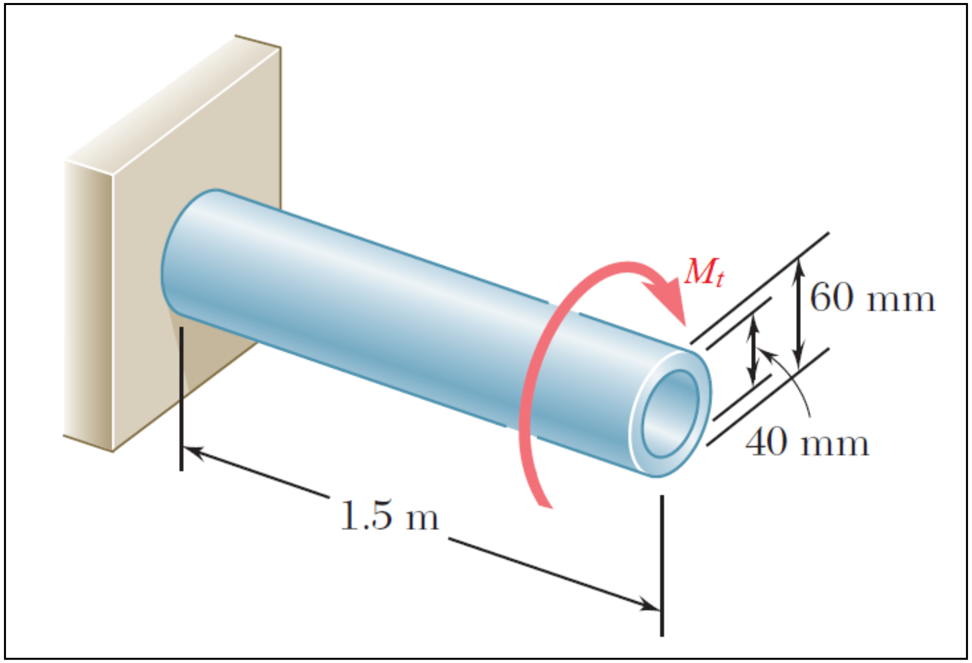
innovate achieve lead

Problem

A hollow cylindrical steel shaft is 1.5m long and has inner and outer diameters respectively equal to 40 and 60mm.

(a) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120Mpa?

(b) What is the corresponding minimum value of the shearing stress in the shaft?

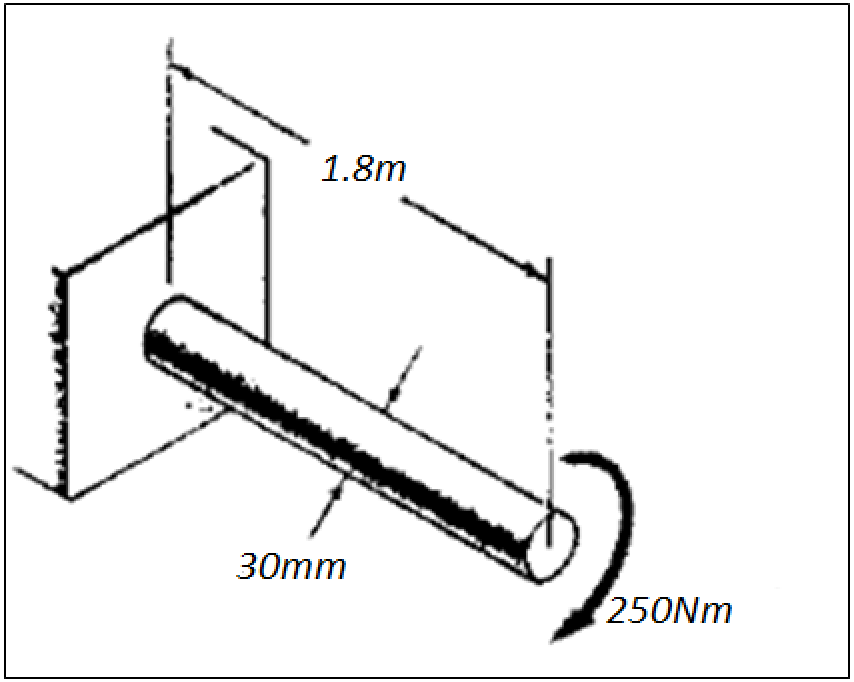


Torsion

Problem

a) For the solid steel shaft shown (G = 80GPa), determine the angle of twist and *A*.

b) Solve part a), assuming that the steel shaft is hollow with a 30mm outer diameter and a 20mm inner diameter.



References

1. Introduction to Mechanics of Solids by S. H. Crandall et al (In SI units), McGraw-Hill